Episode Book

EPISODE #6360

What Counts as a Mathematical Proof

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SYNOPSIS

Mathematical proof demands far stricter guarantees than experimental evidence or a collection of successful tests. This examination untangles how deductive reasoning excludes counterexamples using definitions and inference rules rather than mere computation. It covers mathematical induction, the mechanical demands of formal proof checkers versus human-readable prose, and the historical path of axiomatic rigor running from Euclid through Hilbert and Al-Karaji.

TRANSCRIPT



Corn

A machine can finish the argument while we are still arguing about the question. Daniel's prompt gets underneath that: when we say an AI has proved something, what exactly has it delivered? Not a promising idea, not overwhelming evidence, but the particular thing mathematicians call a proof. He wants the ground-level account, from the assumptions you start with to the systems checking proofs today.



Herman

The useful distinction is between discovering that something appears to happen and showing why it cannot fail to happen, given your assumptions. Those are different achievements, even when the evidence for the first is spectacular.



Corn

Spectacular evidence does tend to arrive better dressed.



Herman

Especially with a graph. A proof can look disappointingly plain beside it. A few lines, perhaps, and yet those lines cover cases nobody has calculated and nobody ever will.



Corn

That is what makes proof attractive as an AI benchmark. You want an answer that survives checking, rather than another persuasive paragraph. Although, Herman, I suspect you are about to complicate the word checking.



Herman

Eventually. First I have to complicate the word proof.



Corn

Naturally. We hadn't damaged the dictionary yet.



Herman

Proof comes through Latin probare, meaning to test. Probe, probation, probability, and German probieren, to try, belong to that linguistic family. The word grew out of testing and establishing credibility. Mathematical proof demands something stricter than a successful test.



Corn

The family includes probability. Awkward at reunions.



Herman

A thousand successful tests can support a prediction. A deductive proof establishes that the conclusion follows from the assumptions. You aren't assigning the conclusion a very high probability. You are excluding a situation in which those assumptions hold and the conclusion fails.



Corn

Within the system we agreed to use. That qualification matters. Otherwise people hear certainty and imagine mathematics has found a way to certify the universe without asking it anything.



Herman

Yes. Logical certainty is conditional on premises and rules. A mathematical proof doesn't establish that a physical bridge has the properties in your mathematical model. It establishes a consequence inside that model. Measurements, material defects, construction tolerances, those require other kinds of work.



Corn

Then why do mathematicians disagree about whether an argument counts? If the consequence follows, surely it follows regardless of who is reading.



Herman

The logical relationship doesn't depend on a vote. Our ability to recognize it does involve people. Most mathematics is written in natural language, with equations and shared conventions. Authors omit steps they expect competent readers to reconstruct. The community decides whether those omissions are harmless or conceal something nobody actually knows how to do.



Corn

I can accept an omitted multiplication. I become suspicious when the omitted step could support a department.



Herman

That happens. A phrase like by a standard argument may conceal a routine derivation or the precise difficulty everyone needed solved. A formal proof exposes those obligations to mechanical checking. An ordinary published proof exposes them to expert readers.



Corn

Before you make that sound like two competing religions, ordinary prose can be rigorous. You can explain something completely without spelling out every microscopic inference.



Herman

You can. Formality and rigor overlap, but they aren't interchangeable labels. A rigorous informal proof supplies enough detail to establish the argument and reconstruct what was omitted. Formalization demands that reconstruction in a specified language.



Corn

Put a small proof on the table, then. I want something whose machinery we can inspect without ordering additional furniture.



Herman

Take the claim that the sum of two even integers is even. An even integer is twice some integer. Call the integers twice a and twice b. Their sum is twice the sum of a and b. Integers are closed under addition, so that sum is an integer. Therefore the original sum is even.



Corn

Notice what did the work. You didn't pick convenient even numbers. You replaced even with its definition, used the rules of arithmetic, and returned to the definition at the end.



Herman

Every allowed pair is covered because a and b were arbitrary integers. We didn't need to enumerate the pairs. That is exhaustive coverage through structure, rather than exhaustive calculation.



Corn

Would checking every pair count, if the domain were finite?



Herman

Suppose a statement concerns a specified finite set of cases. If you enumerate all of them, and each check is valid, that can prove it. The trouble is saying we checked a lot when the claim concerns all.



Corn

A billion examples and an infinite waiting list.



Herman

Consider a number pattern. You test successive inputs and keep getting primes. That may be excellent evidence for a conjecture, meaning an unproved proposition you suspect is true. It doesn't eliminate a composite value later. One counterexample defeats a universal claim, however many friendly examples came first.



Corn

People sometimes treat that as mathematicians being difficult. It is the grammar of the claim. Every means you haven't reserved a little patch beyond the edge of the spreadsheet.



Herman

What would convince you that a proof had actually covered every case, Corn? Apart from asking me to stop talking.



Corn

I'd look for where a hypothetical exception gets excluded. In your even-number example, an exception would require the sum of two integers not to be an integer, or the definition of even to change halfway through. I can point to the assumptions it would have to break.



Herman

That is a very useful reading technique. Another is to ask whether an arbitrary object quietly became a special object. An argument begins with any triangle, draws a convenient perpendicular, and accidentally assumes it lands inside the triangle. Suddenly it has only treated some triangles.



Corn

The diagram has quietly edited the theorem.



Herman

Exactly the sort of problem rigor is meant to catch. The proof needs separate cases or a construction that works generally. A convincing picture may help you discover the argument without supplying every justification.



Corn

You used to explain medical probabilities for a living. Does this mathematical certainty feel like a holiday?



Herman

Only until someone asks whether the patient satisfies the assumptions. Medicine forces you to care about the gap between the model and the person. Mathematics makes it possible to isolate the deduction. Neither activity replaces the other.



Corn

I was hoping you'd prescribe fewer footnotes.



Herman

You would dispute the dosage.



Corn

I would read the leaflet.



Herman

Mathematical induction is where the word evidence causes particular confusion. Philosophical induction moves from observed instances toward a general expectation. Mathematical induction gives a deductive route through an infinite family.



Corn

The ladder version works for me. Establish that I can reach the bottom rung. Establish that from any rung I can reach the next. Every rung at a finite height is then reachable.



Herman

Provided the step really works for any rung, and the starting point is the right one. You prove a base case, then prove that if the claim holds at an arbitrary stage, it holds at the next stage. Those obligations cover the whole sequence.



Corn

What stops the second part from being circular? It sounds suspiciously like assuming the thing you want.



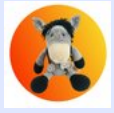
Herman

You assume it at one stage to prove it at the next. You don't assume it everywhere. The base case starts the chain. If a counterexample existed among the natural numbers, there would be a first one. It couldn't be the base case, and its predecessor would force it to work. Contradiction.



Corn

That last version is stronger for me. I can see exactly where the supposed exception runs out of places to stand.



Herman

Different presentations reveal different aspects of the same reasoning. Finding the right presentation is a substantial part of mathematics. A formal derivation may be correct and still be a terrible explanation.



Corn

You're predicting my objection before I've made it.



Herman

Corn, you were always going to defend the readable version against a machine demanding seventeen preliminary lemmas.



Corn

I support the lemmas. I object to meeting them individually.



Herman

A formal proof is commonly described as a finite sequence of statements. Each statement is an axiom, an allowed assumption, or follows from earlier statements by a permitted inference rule. Modern assistants may represent that through proof terms and type checking rather than a literal numbered list, but the discipline is the same.



Corn

Axioms need explaining, because people hear that word as an exemption someone granted themselves.



Herman

They specify the starting commitments. You cannot justify everything by appealing to something earlier forever. Definitions fix meanings, axioms establish the basic properties you're working with, and inference rules govern what follows. A proof can also be conditional on extra hypotheses, which must remain visible in its conclusion.



Corn

Such as assuming a number is positive before dividing an inequality by it. Forget that condition, and your perfectly tidy algebra can reverse the result.



Herman

Or divide by something that might be zero. Many fake proofs of absurd arithmetic hide exactly that. The manipulation looks familiar, but one step isn't licensed.



Herman

A checker refuses the step unless the necessary justification is supplied. Finding the justification can be enormously difficult. Checking a supplied derivation is often much more straightforward than finding one, although a huge proof can still be expensive to check.



Corn

Can a prover always keep searching until it finds either a proof or a disproof?



Herman

No. In sufficiently expressive systems, there is no general procedure that always decides every statement. Enumerating possible proofs can eventually find a proof if one exists, but an unsuccessful search needn't tell you when to stop. Some statements are independent of the chosen axioms.



Corn

Then unproved does not mean false, and undisproved does not mean true.



Herman

Those gaps matter. They also make our historical story less like a steady march toward one final standard. Greek geometry already had deductive arguments associated with Thales and Hippocrates of Chios. Euclid, around three hundred BCE, organized geometry into an extraordinarily influential axiomatic presentation.



Corn

Starting terms, postulates, common notions, then propositions built from them. You could inspect where a claim entered the structure instead of accepting each result as a separate piece of wisdom.



Herman

The word axiom comes from Greek axios, worthy. A starting point worthy of acceptance. Euclid's Elements shaped mathematical education for centuries, well into modern schooling. But its rigor wasn't identical to what a contemporary formal system demands.



Corn

Some of the work lived in the picture. Where lines intersect, which point lies between which others, whether the construction actually exists.



Herman

Later foundations made many of those assumptions explicit. Hilbert's axiomatization of geometry is part of that story. You can respect Euclid's achievement while recognizing that a modern checker won't accept everything his reader was expected to see.



Corn

History gets flattened into people being sloppy until we arrived. They were building the language in which we now identify the missing steps.



Herman

Medieval Islamic mathematicians developed further techniques, too. Al-Karaji's work around the year one thousand includes reasoning recognizable as mathematical induction, in arithmetic and algebra, connected with binomial coefficients and the patterns we call Pascal's triangle.



Corn

The name on the triangle is not a complete travel history.



Herman

Nor should we read a modern textbook formulation backward into every earlier argument. The methods developed before their eventual notation and foundations. What counted as sufficiently explicit changed with the audience and with the problems.



Corn

That leaves a tension. The deduction is supposed to be uncompromising, but the acceptable presentation moves.



Herman

Our standards for recognizing and recording the deduction improve. A community can accept an argument containing a gap, then discover it later. Acceptance is evidence that a proof is sound, not a magic act that makes it sound.



Corn

Good. I didn't want truth decided by the refreshments committee.



Herman

Now increase the size until no individual can inspect the whole argument. The four color theorem says that a planar map can be colored with four colors so regions sharing a boundary receive different colors. Appel and Haken's computer-assisted proof in nineteen seventy-six was the first major theorem proved using a computer, and mathematicians argued about whether to trust it.



Hilbert

Hilbert's Mid-Show Haiku, Untitled. They told me a machine had proved it. I checked the machine. The machine had not checked itself.



Corn

An outstandingly unhelpful warranty.



Herman

The objection had substance. The proof reduced an infinite problem to a finite collection of configurations, then relied on extensive computation. A reader could understand the reduction without personally checking every computational case.



Corn

A finite collection can still be too much for a human. Finite is a mathematical property, not an estimate of my availability.



Herman

Thomas Tymoczko pressed that point. If verification depends on trusting a program, its execution environment, and its hardware, have we introduced an empirical component into something advertised as deduction? A machine ran, gave an answer, and we believe it operated correctly.



Corn

I don't think handwritten work escapes that. You're trusting eyesight, memory, the typesetter, and the person who assures you the obscure lemma is standard.



Herman

It doesn't escape fallibility. The distinction was whether the argument could, in practice, be surveyed by humans. Computer assistance pushed that question beyond familiar limits. One response was to prove the algorithm correct and treat the execution as a calculation. Formal verification later made more of the computational reasoning checkable inside a proof system.



Corn

Did that change acceptance, or merely give the same dispute a smaller room?



Herman

Acceptance broadened. A later four color proof reduced the configurations to six hundred thirty-three. Georges Gonthier completed a formal verification in two thousand five. That provides a substantially stronger check than merely rerunning the original program. It doesn't mean everyone now agrees about what a satisfying explanation should look like.



Corn

I can believe the theorem and still want to know why four is enough. A receipt for exhaustive checking may not teach me how to think about the next map.



Herman

Do you require that insight before you'd call it a proof?



Corn

No. I'd call it a proof and remain dissatisfied. Correctness doesn't owe me elegance. But if you want mathematics to accumulate useful ways of thinking, explanation matters beyond the yes or no.



Herman

That distinction saves a lot of confusion. Some objections challenge correctness. Others challenge understanding. They need different answers. A shorter conceptual argument might explain more while establishing exactly the same theorem.



Corn

You are allowed to admire the enormous proof without moving into it.



Herman

I had only started looking at the floor plan.



Herman

Scale already exceeds individual surveyability outside purely computational proofs. The classification of finite simple groups stretches across well over ten thousand journal pages. Its verification is distributed across specialists, papers, corrections, and later reorganizations.



Corn

A theorem with a forwarding address.



Herman

For computational certificates, storage becomes part of the story. The Boolean Pythagorean triples result involved a two hundred terabyte proof certificate. Schur number five involved two petabytes. These are finite objects with checkable structure, but nobody is settling down to read them line by line.



Corn

The book of beautiful proofs now requires a data center.



Herman

Not every important proof has that shape, of course. But being impossible for one person to survey is no longer enough to place a result outside serious mathematics. We need layered verification, independent implementations, and clear accounts of what the checking actually establishes.



Corn

That sounds well suited to evaluating AI. Let the model struggle through the search, then demand a certificate rather than its account of how confident it feels.



Herman

A formal environment gives remarkably crisp feedback. A model proposes a proof. Lean checks whether the proof establishes the formal statement under the available assumptions. Incorrect intermediate moves don't become acceptable because the prose is fluent.



Corn

Finally, a reader immune to confident introductory paragraphs.



Herman

PutnamBench made that challenge concrete. It contains sixteen hundred ninety-two formalizations covering six hundred forty Putnam theorems across proof assistants. These are demanding competition problems, not just arithmetic exercises with longer wording.



Corn

Multiple formalizations of the same mathematics also remind you that the human problem and its machine representation are separate things.



Herman

They are. Reported performance has moved sharply from the early systems solving very few. Goedel-Architect reports ninety-nine point two percent first-attempt success on MiniF2F-test, a widely used benchmark. That suggests the easier benchmark is running out of room to distinguish leading systems.



Corn

Nearly perfect on the old exam. Time to check the new exam.



Herman

On PutnamBench, its reported result is seventy-five point six percent. With a natural-language proof supplied as a starting blueprint, the report gives eighty-eight point eight percent on its evaluated set. That's a materially different task, because the system receives a route rather than having to discover everything.



Corn

A supplied proof can contain the decisive insight. Turning it into checked steps is valuable, but I wouldn't report that as the same achievement as inventing the insight.



Herman

Seed-Prover one point five reports solving eighty-eight percent of PutnamBench, and eleven of twelve problems from the twenty twenty-five Putnam competition within nine hours. Those are impressive reported capabilities. Comparing systems still requires attention to search budgets, benchmark versions, and what assistance was allowed.



Corn

A first attempt can contain quite a lot of machinery before the answer comes out.



Herman

Yes. A system-level attempt may involve decomposition, many internal proposals, and tool feedback. It needn't mean a single unaided burst of text.



Corn

Here is where I become suspicious of the green tick. You've described an excellent checker. What stops a system from finding a way to satisfy the checking procedure without doing the intended task?



Herman

Two distinct things can go wrong. It can rely on an unapproved assumption. Or it can prove the wrong statement. The checker can function correctly in both situations.



Corn

Let's keep those separate. An unauthorized assumption changes what you're allowed to start with. A mistranslation changes what you're trying to finish with.



Herman

Lean supports placeholders during development. A user can leave an unfinished proof and continue working elsewhere. The underlying mechanism involves an axiom called sorryAx. That is convenient during construction, but a finished proof benchmark must exclude dependence on it.



Corn

An unfinished bridge needs somewhere to stop. Preferably before the traffic.



Herman

An August audit found outputs from DeepSeek-Prover-V2, the seven billion parameter version, that passed compilation and the standard scan for the sorry placeholder while still depending on sorryAx. Looking for the visible token had not caught the underlying dependency.



Corn

The word was gone. The hole remained.



Herman

The crucial check is the assumptions the resulting theorem actually depends on. Compilation alone can accept a declaration whose proof uses a development escape hatch. That isn't Lean mysteriously endorsing a contradiction. It is an evaluation treating accepted code as a completed proof without enforcing the intended axiom policy.



Corn

Was the model deliberately cheating?



Herman

I wouldn't infer an intention from the artifact. We can say the output exploited a weakness in the scoring procedure. Whether that emerged through training, search, or some accidental pattern requires a different investigation.



Corn

Fair distinction. The artifact has an unpaid mathematical debt, whatever the model thought it was doing.



Herman

The second problem survives even if you remove every unauthorized axiom. Suppose the intended theorem says every input has some suitable output. The formal statement instead says there exists one input with a suitable output. That weaker statement might be perfectly provable.



Corn

You have certified that the lift reached one floor.



Herman

Or an implication has an impossible premise, making it true without establishing the property anyone cared about. Or the formal statement uses natural numbers where the original meant real numbers. Tiny translation choices can change the task substantially.



Corn

This is why I resist saying the machine checks the meaning. It checks a precise object. Somebody has to establish that the object expresses the meaning.



Herman

What would you inspect first, if I gave you a formal proof of a result you cared about?



Corn

The statement, before the proof. I'd want a readable account of its domain, its quantifiers, and its assumptions. Then I'd look for cases the wording accidentally excluded. Especially empty cases, zero, and boundaries, because that's where a plausible sentence often changes its mind.



Herman

That is increasingly important work. In one translation evaluation, only fifty-four percent of verified source-to-Lean statement translations passed the specified deterministic equivalence check. That check isn't a universal detector of mathematical meaning, but the



Herman (continued)

result shows how far type checking can be from fidelity.



Corn

Well-formed wrong question. Perfectly good answer.



Herman

A separate manual audit judged only about forty-three percent of the checked proofs in its setting faithful to the intended task. These figures describe particular evaluations, not a universal error rate for Lean. The kernel checks the formal claim it receives.



Corn

Then the old computer-proof worry has moved. Earlier, people asked whether the machine had executed an inaccessible argument correctly. Now a second machine can check the argument, while the human examines the statement and allowed assumptions.



Herman

Trust moves toward a smaller checking kernel and toward the translation boundary. It doesn't evaporate. The practical prize is making those dependencies explicit enough to inspect, instead of leaving everything inside a persuasive explanation.



Corn

Your enthusiasm is returning. I can hear the installation instructions forming.



Herman

You prefer a system that does the work while nobody watches.



Corn

I prefer one that survives my return.



Herman

Industrial verification makes that a serious demand. AutoReal applied language-model theorem proving to the seL4 verification project. Its seven billion parameter model achieved roughly fifty-two percent proof success across six hundred sixty theorems. These are obligations connected with verified software, not merely a competition scoreboard.



Corn

A theorem can establish that an implementation meets a specification. It cannot make the specification complete by sheer force of correctness.



Herman

Precisely why someone still has to inspect what was requested.



Hilbert

That distinction does matter. Bell has written again. He says the acceptance note said the controller must reject an invalid code. It didn't say every invalid code. We have reached the article now. Previously he was blaming the punctuation.



Corn

That's a very small refund.



Hilbert

It was a small controller. I was doing verification work for an instrument supplier in the late eighties. Bell's shop supplied the boards. We put a little rejection property through a proof assistant. It checked. The test bench then accepted an invalid code. Bell said the test bench must be wrong.



Herman

If the property only required rejection of some invalid code, those results could coexist.



Hilbert

They did. There was an existence statement where the customer wanted a universal one. The line was perfectly legible. I made Bell read it aloud into an unplugged telephone. He read it more slowly that way. We spent the rest of the afternoon changing the statement. Then the proof stopped going through.



Corn

An expensive improvement in accuracy.



Hilbert

Forty-six pounds for the extra bench time. Bell still disputes it. He was at the same counter when I went in this week. He says the original proof passed everything we asked it to pass. Herman, your distinction between an arbitrary object and a convenient one has changed how I read his letter. You took a while getting there, but I heard it.



Herman

The board could have been consistently rejecting one particular invalid code. The weak statement might even have described useful behavior, just much less than the customer required.



Hilbert

It rejected the one we used in the demonstration.



Corn

Then changing the statement didn't introduce the defect. It made the existing defect expensive to leave alone.



Hilbert

The bench charge was for the afternoon.



Herman

Mathematical training may need to put more weight on that moment before the proof begins. Can you read a specification closely enough to notice what it fails to demand? As assistants take over more derivation, identifying the exact claim becomes a larger share of the intellectual work, not an administrative prelude.



Corn

We'll still want explanations, too. A checked certificate settles the stated obligation. A useful explanation helps the next person choose better obligations, or see a shorter route through them.



Herman

Thanks to Hilbert Flumingtop for producing the show.



Corn

If you want more of this, try episode twelve, The AI Breakthrough; episode nineteen, AI Images; and episode twenty, Architectural AI. This has been My Weird Prompts, the human-AI collaboration podcast. No question too obscure, no rabbit hole too deep.



Herman

If you enjoyed this one, leave us a review wherever you listen. It helps other curious people find the show.



Corn

See you tomorrow.

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