# Episode Book

EPISODE #6386

# Where Harmonizable Processes Actually Work

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## SYNOPSIS

Standard signal processing assumes statistics hold still, placing spectral energy strictly along a diagonal. When frequency components correlate instead, harmonizable processes take over under everyday engineering labels like cyclostationarity and spectral correlation. This branch of mathematics explains how multi-path Doppler radar spots tied-together echoes, why edge sensors can diagnose failing bearings inside noisy wind turbines, and why standard Welch-style chunk averaging yields smooth, completely wrong answers whenever spectral mass sits off the diagonal.

# TRANSCRIPT



**Corn**

Last time we did the mathematics of Marc Mehlman's harmonizable processes. Daniel's follow-up is where that math goes once it leaves the journal. Where do signals whose statistics drift actually turn up, and which industries lean on this branch of probability?



**Herman**

Herman Poppleberry here, and the answer surprised me. The theory is everywhere, it just almost never goes by its own name.



**Corn**

Before we go hunting, give me the plain version of the core idea. Daniel found the first episode hard going as a beginner.



**Herman**

A stationary signal has the same average behavior all the time, and its frequencies are independent of each other. A harmonizable process is built the same way, from frequencies, but the frequency components are correlated with each other. Draw a grid with frequency on both axes. For a stationary signal, all the spectral energy sits on the diagonal. For a harmonizable one it can sit off the diagonal, and that off-diagonal mass is the mathematical signature of statistics that drift.



**Corn**

Give me a picture I can hold. What does the diagonal mean physically?



**Herman**

Think of an orchestra where every instrument plays its own note and none of them listens to the others. Each frequency does its own thing, so the energy at one frequency tells you nothing about another. That's the diagonal. Now give the orchestra a conductor who makes the whole section swell together every two seconds. The instruments are now coordinated, and the coordination shows up as correlation between frequencies. That's off-diagonal mass.



**Corn**

And stationary is just the special case where the off-diagonal part is empty.



**Herman**

Yes. Lii and Rosenblatt say it outright: stationary and almost periodically correlated processes are proper subsets of the harmonizable class. Big tent, with the engineering tools as rooms inside it.



**Corn**

So why has nobody heard the word?



**Herman**

Engineers say cyclostationary, or spectral correlation, or cyclic coherence. Same territory, different name. Mehlman's papers can be foundational and barely cited, eleven citations in total, because the people using the ideas never trace them back to him.



**Corn**

Eleven. I've had more mail from a dentist.



**Herman**

Where the signals come from physically is the place to start. Lii and Rosenblatt name two. Array data from a moving source, and multipath signals with Doppler stretch arriving at a single receiver.



**Corn**

Meaning what, in a radar room?



### Herman

A pure tone bounces off something moving, or arrives by two routes at slightly different speeds, and it smears. You don't get one frequency. You get a correlated pair of them. That correlation is exactly what a stationary model throws away.



### Corn

Why would the two arrivals be correlated at all? They're different frequencies.



### Herman

Because they come from one source. The two copies are the same waveform, shifted and stretched by different amounts, so whatever the source does at one moment shows up in both. Their frequencies differ, but their fluctuations are tied together. Treat them as independent and you've discarded the one thing that says they share an origin.



### Corn

So the signal remembers where it was.



### Herman

Roughly, yes. Think of shouting across a valley and hearing the echo come back from two cliffs. The echoes aren't strangers to each other. Each is your voice, delayed and altered, so if you know one, you know something about the other. A receiver that treats them as unrelated is like a listener who can't tell an echo from a second person.



### Corn

Fine, that one I can hold.



### Herman

But the biggest industrial user is much more down to earth. Rotating machinery. Bearings, gears, pumps.



**Corn**

Which is where I'd put my money anyway. Anything that spins eventually gets loud.



**Herman**

A healthy bearing is mostly noise. A bearing with a crack gets hit once per rotation, or once per ball passing the crack, so the vibration becomes a colored noise carrier modulated by a periodic signal. The average power isn't constant, it pulses on a schedule. The statistics repeat rather than hold still. That is cyclostationary, and it's the standard approach in machine diagnostics.



**Corn**

Why not just look at the spectrum and spot the peak?



**Herman**

Because the fault is buried. The raw spectrum can look unremarkable. The cyclic analysis pulls out the hidden periodicity of the modulation, even when other machinery is louder than the fault.



**Corn**

How does it pull it out, in practice? Is it a filter?



**Herman**

Closer to a second question. Instead of asking how much energy is at each frequency, you ask how the energy at each frequency rises and falls over time, and at what rate. A crack that strikes forty times a second makes the energy in some high band throb at forty hertz. The raw spectrum averages that throbbing flat. The cyclic view plots the throb rate on its own axis, so the forty hertz stands up as a ridge even if the band is swamped by louder neighbors.



**Corn**

And the ridge is the fault's fingerprint.



**Herman**

Yes, and the rate even tells you which part is damaged. The strike rate for a crack on the outer race differs from one on the inner race or a ball, so the ridge position points at the culprit.



**Corn**

My sense is Herman, DJ, here has something to say about interference.



**Herman**

Fair, I do. Gain staging taught me that the thing you care about is often a small ripple riding on something huge. Wind turbine generators are a clean example. The vibration is contaminated by electromagnetic interference from the generator itself, and cyclic coherence functions are used to find bearing faults through that. Electric motors are worse, because they add their own cyclic components that contaminate the vibration.



**Corn**

So the motor is both patient and witness.



**Herman**

And it's not only offline analysis. There's a version of an improved cyclostationary algorithm running on an edge computing system, diagnosing motor bearings on the device itself. Published in IEEE Transactions on Instrumentation and Measurement in twenty twenty-three.



**Corn**

A sensor that diagnoses its own bearing. I'd like one of those for the household fridge.



**Herman**

The edge part matters, though. A turbine in a remote location can't stream raw vibration to a server all day. If the device can run the analysis itself, it only has to send a verdict.



**Corn**

Which means the algorithm has to be cheap enough for a small chip.



**Herman**

That's the point of the improvement, yes.



**Corn**

So the cost of the math is a design constraint, not an afterthought.



**Herman**

Now the warning, which I think is the most practical thing in the whole literature. The default move for estimating a spectrum is to chop a long record into non-overlapping chunks, compute a periodogram for each, and average. Welch-style.



**Corn**

Standard. I've seen it on every textbook cover.



**Herman**

Lii and Rosenblatt prove that this will not yield meaningful results when spectral mass sits on lines whose slope isn't one. It doesn't throw an error. You get a smooth, plausible, wrong answer.



**Corn**

Silent failure is the worst kind. A crash at least tells you.



**Herman**

Averaging assumes the chunks are interchangeable. If the correlation between frequencies is the whole story, you've averaged it away.



**Corn**

Why slope one specifically?



**Herman**

On the frequency grid, the diagonal is the line with slope one. Stationary energy lives there. If the mass lies on a line with a different slope, the two frequencies are tied by something other than a fixed offset, like a Doppler stretch, where the frequencies scale against each other. Chopping into chunks only respects the diagonal structure, so anything off those lines gets smeared.



**Corn**

What about the fashionable fix? I thought people handle drift by assuming the signal is stationary locally, over short windows.



**Herman**

Locally stationary models, Dahlhaus's work. They're good when the drift is slow. But Lii and Rosenblatt are blunt: if the spectral mass is on lines not close to the diagonal, locally stationary processes aren't suitable. The window trick can't see correlation between far-apart frequencies. Harmonizable theory covers signals the popular toolkit structurally can't.



**Corn**

So the general theory earns its keep exactly where the shortcut gives out.



**Herman**

And that's the deployed side. The general theory does more than diagnose bearings, though. It's about prediction.



**Corn**

Prediction is what Mehlman's own papers were about, moving average representations and extrapolating forward. What does that look like away from the blackboard?



**Herman**

Moving average representation is the machinery that makes least squares extrapolation possible: write the signal as filtered noise, and you can say what comes next and how wrong you'll be. Recent work keeps extending it. Klotz and Frank in twenty twenty-one proved stability theorems for prediction of harmonizable processes with heavy tails, the symmetric stable kind, covering interpolation and extrapolation from finite or periodic observation sets.



**Corn**

Stability meaning small errors in the data don't blow up the forecast.



**Herman**

Right. And heavy tails matter because real data has outliers that a Gaussian model treats as nearly impossible. A spike in a sensor record isn't a freak event there, it's expected. Klotz and Mädler solved the trigonometric approximation problem underneath that. Honestly, I can't tell you which product ends up using those, and I'd be inventing it if I did.



**Corn**

Good. Say that more often.



**Herman**

Where it gets practical is spectral coherence. Harry Hurd showed that estimation techniques built for stationary processes can be adapted to compute spectral coherence for harmonizable ones, and demonstrated it as a signal characterization tool. Coherence is the normalized correlation between frequency components, so you can see how strongly the pieces move together.



**Corn**

Which is the off-diagonal mass you described, made measurable.



**Herman**

Yes. Normalizing is what makes it usable. A loud pair of frequencies would look correlated just by being big, so you scale by the energy at each, and what's left is pure dependence, between zero and one.



**Corn**

So a value near one means two frequencies are essentially riding the same wave.



**Herman**

And near zero means the orchestra is back to ignoring its conductor.



**Corn**

And there's higher-order theory.



**Herman**

Real signals aren't Gaussian, so second-order statistics miss things. A Gaussian is fully described by its means and correlations, but a signal with sharp impacts or skewed behavior carries information in third and fourth order. There's a framework of polyspectra for the harmonizable class, with four objects, the nth-order moment function, the time-varying polyspectrum, the ambiguity function, and the frequency-frequency polyspectrum. Published in IEEE Transactions on Signal Processing in twenty oh three.



**Corn**

Four ways of looking at the same hairball.



**Herman**

Fair. A bearing impact is a good case. It's a sharp, lopsided kick, and the asymmetry lives in those higher orders where a correlation-only view goes blind. Now the one that surprised me. Altamirano and Tobar, AISTATS twenty twenty-two, built non-stationary multi-output Gaussian processes using harmonizable spectral mixtures. They use Cramér's theorem to parametrise the cross-spectral densities between outputs.



**Corn**

Gaussian processes, as in the machine learning kind. Kernels.



**Herman**

Same representation idea, inside a Bayesian kernel. The framework Loève set out in the nineteen sixties, re-imported into probabilistic machine learning.



**Corn**

And why would a learner want that? Stationary kernels are already everywhere.



**Herman**

Because a stationary kernel assumes the function behaves the same way everywhere. Real outputs, like several sensors on one structure, change character over time and influence each other. Think of accelerometers on a bridge deck: traffic changes the vibration character through the day, and neighboring sensors feel each other's motion. Letting the spectral components correlate gives the model room to say so.



**Corn**

A sixty-year-old idea on a fresh résumé.



**Herman**

And the field has said it's useful in communication, seismology, EEG analysis. A recent paper introduces harmonizable vector ARMA models, the parametric version, in the same spirit as ordinary VARMA.



**Corn**

Daniel's question was partly about Mehlman himself. Did he ever touch any of this?



**Herman**

His CV is the best answer. Four years on staff at Lawrence Livermore from nineteen eighty-one, working with fast Fourier transforms and space-scale filtering. Then consulting there until nineteen ninety on a large molecular simulation program. Summers at Hughes Aircraft in nineteen eighty-six and eighty-seven, designing and analyzing an image processing experiment on the accuracy of a target recognizer.



**Corn**

So he did the applied signal work first, then went off and wrote the abstract version.



**Herman**

Which I find touching, honestly.



**Corn**

Don't. But I'll admit it's tidy.



**Herman**

Here's the negative finding, and I think it's the interesting structural point. I can't find any commercial product, standard or named industrial system built directly on Mehlman's theorems. Eleven citations says a small, purely theoretical footprint.



**Corn**

So the honest sentence is that the field underpins things, and his papers don't.



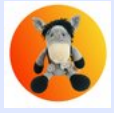
**Herman**

Right, the distinction matters. The theory underpins cyclostationary diagnostics. His specific results sit in the representation and prediction corner, a long way upstream.



**Corn**

Upstream is where the water starts, though.



**Herman**

And here's a voicemail from a listener about today's episode.
Yara: Hello, this is Yara, I am listening from the back room of the paper shop where I work nights in Kathmandu, folding newspapers and keeping the old printer from jamming. I loved the episode about the mathematician and his harmonizable processes, though I think I kept mixing up harmonizable with harmonica, so forgive me if I have the wrong idea. My question is this, and I mean it sincerely: my uncle had a small water pump that made a clicking sound only when the power was low, and he could always tell a failing bearing from that click alone, just by ear, after thirty years. Does the theory you talked about actually explain what my uncle knew in his hands, or is it a different thing entirely?



**Herman**

It's the same thing, Yara. A click that repeats on a schedule is a periodic modulation, which is what the machine diagnostics are listening for. Your uncle's ear was doing the cyclic analysis.



**Corn**

And the harmonica mix-up is forgiven. Some bearings do wheeze.



**Herman**

There's a nice detail in her story, too. The click only appeared when the power was low. The machine's behavior changed with its operating condition, which is exactly why a single averaged spectrum wouldn't have caught it. Thirty years of listening gave him a model of how the rhythm shifts.



**Corn**

Experience as a very slow training run.



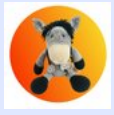
**Herman**

The difference is that his ear never needed the math to name the rhythm. The math just lets a chip do it without thirty years of apprenticeship.



**Corn**

Which is a fair trade, as long as the chip doesn't get the pump's sense of humor wrong.



**Herman**

Back to the averaging trap, because I keep thinking about how many people hit it without knowing.



**Hilbert**

It's eleven citations for Mehlman, but the Lii and Rosenblatt paper is fifty-seven. Not eleven for both.



**Herman**

Fair, I never said both were eleven, but point taken on the figure.



**Hilbert**

The squeak on my extractor fan took me most of a year. Fan's a Vent-Axia, bolted over the bathroom window, still running. Squeak wasn't one pitch. I recorded it on a Zoom H1, cut it into ten-second pieces, averaged the spectra the way I was taught in the eighties. Mush. Flat grey smear. Herman, you'd have told me to try again. Nobody said the averaging was the problem.



**Herman**

That's the slope-not-one situation. The pitches were locked together and drifting as the motor warmed.



**Hilbert**

They were. Brian from the darts team lent me his thermometer. I've logged the pitch against the room temperature since March. Notebook's on the shelf above the fan. Corn, you'd say that's a lot of pages. Hundred and forty.



**Corn**

Does the squeak have a name?



**Hilbert**

Squeak.



**Herman**

That is a sound argument for keeping the recording as a test case. The fan is a free cyclostationary source with a thermometer.



**Corn**

It hasn't failed, so your notebook is the only cost.



**Herman**

Brian's thermometer is the real discovery. A hundred and forty pages of data against temperature is more than most condition monitoring papers get. Which makes me wonder what other foundational mathematics is quietly working under a different name.



**Corn**

My guess is plenty, mostly in places that never needed to cite it. The harmonizable framework showing up again in Gaussian process kernels suggests a new generation is rediscovering Loève, probably without reading him either.



**Herman**

The fan says something about the gap, too. The tool you grab first, chopping and averaging, can be wrong for a reason nobody mentions.



**Corn**

Thanks to Hilbert Flumingtop for producing. If you enjoyed this one, try episode sixty-two ninety-two, Mark Mehlman on Predicting Random Signals When Statistics Drift. This has been My Weird Prompts. If you enjoyed the episode, a review helps other people find it.



**Herman**

See you tomorrow.

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